

A DSGE Model with Housing in the Cointegrated VAR Framework*

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January 18, 2012

Abstract

In order to empirically investigate the assumptions underlying a theoretical DSGE model, the long-run and the short-run structure of the model may be imposed in the framework given by the cointegrated VAR (CVAR) model. By following the method outlined in Juselius and Franchi (2007), I use the CVAR model to investigate the restrictions underlying the DSGE model in Iacoviello (2005), which is a monetary business cycle model that includes housing in order to include effects from the financial accelerator. This yields a common trends representation of the data subject to the theoretical constraints of the DSGE model which can be used to calculate the impulse responses for different shocks. These impulse responses are then compared to the impulse responses presented in Iacoviello (2005). The results given by the estimated CVAR model do not correspond well to the restrictions of the DSGE model. This is shown both by testing the cointegrating relationships implied by the long-run relations, and the response to shocks in the period for which the model is estimated. Imposing theoretical restrictions pertaining to the DSGE model in Iacoviello (2005) on his estimated VAR model yields different impulse responses from various shocks which change some of the main findings of the model. Particularly, imposing long-run homogeneity between housing prices and output and imposing the Fisher relationship which is assumed in the theoretical model seems to yield results which is opposite of what the financial accelerator, which is one of the key motivations behind the model in Iacoviello (2005), suggests. The non-stationarity of inflation and the nominal interest rate may be important reasons for this, together with the non-acceptance of the imposed long-run hypotheses.

Keywords: DSGE, Housing, Cointegrated VAR.

JEL: C32, E32, R21.

1 Introduction

By estimating DSGE models with filtered data, through e.g. the Hodrick-Prescott filter or the band pass filter, the trend and the cyclical component of the data series are separated. Log-linearizing a theoretical model around its steady state, results in a system of linear

*The author thanks Gunnar Bårdsen and Ragnar Nymoen for helpful comments and suggestions.

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equations where the variables express percent deviations from the steady state. By filtering the data, the data in levels are divided into a trend and cycles. The cycle part is then used in order to estimate the DSGE model, and replaces the theoretical variables which express the percentage deviation from steady state. The problem is that the filtered cycle is deviations from an estimated trend, such that there might be a mismatch between the trend component of the data and the theoretical trending relationships in the model.

In addition to the importance of the steady state, an important difference between a theoretical DSGE model and the empirical framework of a VAR model is that the VAR model is linear in the parameters and a DSGE model is often highly non-linear in the parameters. In order to compare the two, the DSGE model needs to be approximated into a linear version. This is often done through log-linearization of the non-linear model in order to ease estimation of the structural parameters.

In order to make use of and test the theoretical steady state, a cointegrated VAR (CVAR) model is estimated in this paper. This approach allows estimation without filtering the data prior to the estimation, since separating the trend from the cycle in the data may be done through the CVAR model by making assumptions on the stochastic trends. The long-run relationships (i.e. the steady states) are also estimated, which allows us to investigate the empirical relevance of the theoretical steady state.

The theoretical model is a log-linearized new-Keynesian model presented in Iacoviello (2005). This model includes nominal debt contracts and collateral constraints tied to housing values in order to include housing in a new-Keynesian model and assess its importance. By using the CVAR model, we may test the long-run restrictions and relationships in the theoretical model along the lines of Juselius and Franchi (2007).

Since the house itself is used as collateral for a house loan, changes in house prices will affect the collateral value of the borrower, and therefore influence to what extent potential borrowers will get a loan. The collateral value is also important for the secondary mortgage market, since increased house prices and hence value of the collateral will make the borrowers able to increase their consumption by taking new loans.

If the consumer prices increase and the loan is set in nominal terms, the payment on the loan will be relatively smaller compared to other consumption goods such that increased prices leads to a wealth effect that can boost consumption. Hence, the negative effect of a cost-push shock on consumption is dampened.

Iacoviello (2005) investigates the impulse response functions for an estimated four-equation VAR model and compare them to impulse response functions of the calibrated theoretical model outlined in his paper. This paper extends the analysis by investigating the impulse response functions of a CVAR model, which will combine the estimated VAR with the theoretical model in Iacoviello (2005).

I start by estimating an unrestricted VAR using the same variables as in Iacoviello (2005), but instead of using variables that are filtered through the Band Pass filter, I use the variables in logs prior to filtering. All of the variables are $I(1)$ ¹ (tested in the appendix), something that is convenient when we consider the estimated VECM (vector error correction model) which expresses a model for the variables in differences and is taking cointegration into account. There should therefore not exist a steady state value for the individual variables according to the data, since they are growing or declining over time.

¹On a 5% significance level, real housing prices seems to be $I(2)$, but for a slightly larger significance level they are $I(1)$ such that $I(1)$ is assumed for the analysis here.

2 Theoretical model

The theoretical model investigated in this paper is, as previously stated, the DSGE model in Iacoviello (2005) which incorporates the financial accelerator by using the housing as an important financial asset. The model is mainly taken from Bernanke et al. (1999), in addition to adding collateral constraints tied to real estate values for firms as in Kiyotaki and Moore (1997) and including nominal debt contracts.

The theoretical steady-state relations implied by the model should be found in the data, and may be investigated through the cointegrated VAR model.

The theoretical steady state relationships can be rearranged in order to get equations that explain a relationship between the observed variables used in the estimated VAR and CVAR model. If the model is correct, we should also be able to find these relationships in the data.

2.1 The agents in the model economy

The DSGE model of in Iacoviello (2005) contains an economy populated by patient households (their variables are denoted with a prime as superscript), impatient households (denoted with a double prime), entrepreneurs (denoted without any primes), retailers and a central bank.

Patient households maximize a lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln c'_t + j \ln h'_t - (L'_t)^\eta / \eta + \chi \ln(M'_t/P_t)),$$

where $\beta \in (0, 1)$ is the discount factor, c'_t is consumption at time t , h'_t is the holding of housing, L'_t are hours work and M'_t/P_t are money balances divided by the price level. j is a preference parameter related to housing, η measures the labor supply aversion and χ measures the preference for holding money, s.t.

$$c'_t + q_t \Delta h'_t + R_{t-1} b'_{t-1} / \pi_t = b'_t + w'_t L'_t + F_t + T'_t - \Delta M'_t / P_t,$$

where $q_t \equiv Q_t/P_t$ denotes the real housing price, $w'_t \equiv W'_t/P_t$ the real wage and $b'_t \equiv B'_t/P_t$ denotes the amount borrowed in real terms where they pay back $R_{t-1} B'_{t-1}/P_t$ where R_{t-1} is the nominal interest rate on loans between period $t-1$ and t . $\pi_t = P_t/P_{t-1}$ is the gross inflation rate and F_t are lump-sum profits received from the retailers.

The impatient households are assumed constrained and maximize

$$E_0 \sum_{t=0}^{\infty} (\beta'')^t (\ln c''_t + j \ln h''_t - (L''_t)^\eta / \eta + \chi \ln M''_t / P_t)$$

where $\beta'' < \beta$ which guarantees that they will hit the borrowing constraint in equilibrium. This is maximized s.t.

$$c''_t + q_t \Delta h''_t + R_{t-1} b''_{t-1} / \pi_t = b''_t + w''_t L''_t + T''_t - \Delta M''_t / P_t - \xi_{h,t},$$

where $\xi_{h,t}$ is a housing adjustment cost such that households are not perfectly mobile. The borrowing limit is given as

$$b''_t \leq m'' E_t(q_{t+1} h''_{t+1} / R_t),$$

where m'' is the loan-to-value ratio for the impatient households.

Entrepreneurs produce an intermediate good Y_t according to a Cobb-Douglas constant returns-to-scale technology

$$Y_t = A_t K_{t-1}^\mu h_{t-1}^\nu L_t'^{\alpha(1-\mu-\nu)} L_t''^{(1-\alpha)(1-\mu-\nu)},$$

where A_t is random, K is capital and L' and L'' are the labor from the patient and the impatient households, respectively. They maximize

$$E_0 \sum_{t=0}^{\infty} \gamma^t \log c_t$$

where $\gamma < \beta$, s.t.

$$Y_t/X_t + b_t = c_t + q_t \Delta h_t + R_{t-1} b_{t-1}/\pi_t + w_t' L_t' + w_t'' L_t'' + I_t + \xi_{e,t} + \xi_{K,t}$$

$$b_t = m E_t(q_{t+1} h_t \pi_{t+1}/R_t)$$

and technology

The retailers buy intermediate goods Y_t from entrepreneurs, differentiate them and sell them. This imposes sticky prices in the model and the New Keynesian Phillips curve in the log-linearized model.

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1} - \kappa \hat{X}_t + \hat{u}_t,$$

where $X_t \equiv P_t/P_t^w$ is a markup of final over intermediate goods since P_t^w is the wholesale price and P_t is the final good price index. The hat over a variable implies the percentage deviation from its steady state, and $\hat{u}_t$ expresses an inflationary shock.

The central bank follows a Taylor-type interest rate rule, which is log-linearly expressed as

$$\hat{R}_t = (1 - r_R)(1 + r_\pi) \hat{\pi}_{t-1} + r_Y(1 - r_R) \hat{Y}_{t-1} + r_R \hat{R}_{t-1} + \hat{e}_{R,t},$$

where r_R , r_π and r_Y are coefficients in the Taylor rule pertaining to past interest rate, inflation and output in deviation from steady state, respectively. $\hat{e}_{R,t}$ corresponds to a monetary policy shock.

2.2 First-order conditions

The first-order condition for debt for the patient households that maximizes their lifetime utility function is

$$\frac{1}{c_t'} = \beta E_t \frac{1}{c_{t+1}'} \frac{R_t}{\pi_{t+1}}, \quad (1)$$

which in steady state becomes

$$\frac{1}{\beta} = \frac{R}{\pi} \Leftrightarrow \beta = \frac{\pi}{R}. \quad (2)$$

It is also assumed that inflation, π , equals unity in steady state, so that

$$\frac{1}{\beta} = R. \quad (3)$$

The nominal interest rate also is stationary in the long run.

This relationship also enters into entrepreneur's first-order condition for housing when they maximize their lifetime utility which is

$$\frac{q_t}{c_t} = E_t \left(\frac{\gamma}{c_{t+1}} \left(\nu \frac{Y_{t+1}}{X_{t+1} h_t} + q_{t+1} \right) + \lambda_t m \pi_{t+1} q_{t+1} \right) - \phi_e E_t \left[\frac{q_t}{c_t} \frac{\Delta h_t}{h_{t-1}} - \gamma \frac{1}{2} \frac{q_{t+1}}{c_{t+1}} \frac{h_{t+1}^2 - h_t^2}{h_t^2} \right] \quad (4)$$

which in steady state becomes

$$\frac{q}{c} = \gamma \frac{q}{c} + \frac{1}{c} \gamma \nu \frac{Y}{X} \frac{1}{h} + \frac{\gamma}{c} \lambda m \pi q \quad (5)$$

We also have the following first-order condition for debt for the entrepreneurs:

$$\frac{1}{c_t} = E_t \gamma \frac{R_t}{c_{t+1} \pi_{t+1}} + \lambda_t R_t \quad (6)$$

which in steady state becomes

$$\lambda c = \frac{1}{R} - \frac{\gamma}{\pi} \quad (7)$$

Inserting this into the steady state of the first-order condition for housing, (5), we get

$$\frac{q}{c} = \gamma \frac{q}{c} + \frac{1}{c} \gamma \nu \frac{Y}{X} \frac{1}{h} + \frac{\gamma}{c} m \pi \frac{q}{c} \left(\frac{1}{R} - \frac{\gamma}{\pi} \right) \quad (8)$$

or simplified to

$$\frac{q}{Y} = \frac{\gamma \nu}{1 - \gamma - (\frac{\pi}{R} - \gamma) m} \frac{1}{X h} \quad (9)$$

We should also insert for the housing stock, h , which depends on R and π and a combination of various parameters as shown in Iacoviello (2008) where an expression for h is found after calculating various first-order conditions for the agents in the model economy. Using various equations, we obtain an expression for the steady state of the housing stock. Inserting this yields the following steady state relationship:

$$\begin{aligned} \frac{q}{Y} = & \frac{j'}{1 - \beta} \left(s' + (1 - \beta) \left(m \frac{\gamma \nu}{1 - \gamma - (\beta - \gamma) m} \frac{1}{X} \right. \right. \\ & \left. \left. + m'' \frac{j''}{1 - \beta'' - m''(\beta - \beta'')} \frac{s''}{1 + (1 - \beta) m'' \frac{j''}{1 - \beta'' - m''(\beta - \beta'')}} \right) \right) \\ & + \frac{j''}{1 - \beta'' - m''(\beta - \beta'')} \frac{s''}{1 + (1 - \beta) m'' \frac{j''}{1 - \beta'' - m''(\beta - \beta'')}} + \frac{\gamma \nu}{1 - \gamma - (\beta - \gamma) m} \frac{1}{X} \end{aligned} \quad (10)$$

We would like to express a linear relationship between house prices, output, the nominal interest rate and inflation. This may be done by taking logs on each side of (10) and a first-order Taylor approximation such that $\beta = (\pi/R)$ is also expressed linearly. This yields as shown in appendix C

$$\ln q = \ln Y + a + b(\pi/R), \quad (11)$$

where a and b consists of a combination of a number of different parameters. (11) suggests a linear combination between log of house prices, log of output and (π/R) . Since π is the gross inflation rate and R is the gross nominal interest rate, (π/R) is the inverse of the gross real interest rate.

The steady state relationship expressed in (11) may be related to the statistical concepts of the CVAR model, through the cointegrating relations. If the data for the variables expressed in (11) are non-stationary, a cointegrating relation should explain a stationary relationship between non-stationary variables.

In the data set used for the estimation, the nominal interest rate and the inflation rate differs from its theoretical counterparts π and R since the theoretical model uses the nominal interest rate and inflation plus unity, whereas the data set does not. However, the approximation

$$R_{real} = R^{data} - \pi^{data} \simeq \frac{1 + R^{data}}{1 + \pi^{data}} - 1 = \frac{R}{\pi} - 1, \quad (12)$$

where R_{real} denotes the real interest rate (not in gross terms). This may be used such that we can rewrite (11) in order to match the variables in the data set and expressing the real interest rate

$$\ln q = \ln Y + a + b((\pi^{data} - R^{data}) + 1). \quad (13)$$

This may be modified to

$$\ln q = \ln Y + \alpha - b(R^{data} - \pi^{data}), \quad (14)$$

where $\alpha \equiv (a + b)$. This yields an inverse housing demand equation in which housing demand depends positively on aggregate output and negatively on the real interest rate. This implies that we should have $b > 0$. This condition is fulfilled, since $b = 1.28$ assuming $j = j' = j'' = 0.1$ and using the estimated parameter values in Iacoviello (2005).

2.3 Theoretical implications for the cointegrated VAR model

In order to relate (14) to the cointegrating relations in the CVAR model, the choice of rank for the model determines how we should make the connection between the theoretical and the statistical model. The rank determines the number of cointegrating relations, which implies how many stationary relationships that exists in the data. According to the theoretical model and the relationship in (14), we have a stationary relationship between all four observable variables. If we consider output and housing prices to be non-stationary and driven by the same stochastic trend, and the nominal interest rate and inflation may considered to be stationary, as shown in (2) and (3), we may set the rank to $r = 3$ corresponding to the three following cointegrating relations:

$$\begin{aligned} \ln q_t - \ln Y_t &\sim I(0) \\ R_t &\sim I(0) \\ \pi_t &\sim I(0), \end{aligned} \quad (15)$$

which implies one common stochastic trend which drives housing prices and output.

If we consider the nominal interest rate and inflation to be non-stationary, we obtain a different interpretation of the theoretical long-run relationship. From (9), we see that if the nominal interest rate and the inflation rate is non-stationary, there should be a long-run relationship between all of the four observable variables that is stationary in the long run. This indicates three common driving trends for the model. From (9), we see that there is a relationship between the four variables. All of these variables are assumed non-stationary, and in the long-run this relationship should be stationary even though the variables individually are non-stationary.

We also see from (2) that there should be a stationary relationship between inflation and the nominal interest rate. However, if this relationship is not stationary in the long run, we only have one long-run relationship which is given by (14) as an expression where housing prices are determined by output and the real interest rate such that we have an inverse demand function for houses. This yields the following cointegrating relation when setting the rank to $r = 1$:

$$\ln q_t - \ln Y_t - \alpha + b(R_t - \pi_t) \sim I(0). \quad (16)$$

If there is a stationary long-run relationship between the nominal interest rate and inflation, this could constitute a separate cointegration vector such that the rank should be set to $r = 2$, yielding

$$\begin{aligned} \ln q_t - \ln Y_t &\sim I(0) \\ R_t - \pi_t &\sim I(0). \end{aligned} \quad (17)$$

The hypotheses pertaining to (15), (16) and (17) may be tested statistically through the cointegrating relationships in the estimated CVAR model.

2.4 MA representation

The MA representation of the VAR model is useful for formulating assumptions underlying the DSGE model in Iacoviello (2005) as testable hypotheses within the VAR.

The DSGE model is driven by output, which is driven by productivity (A_t). The productivity is assumed to follow an AR(1) process. The model in Iacoviello (2005) is also extended with AR(1) dynamics in the short-run changes of output, housing prices, inflation and the interest rate.

If we assume that total factor productivity, a_t , drives the model, we have the following conceptual MA representation of the DSGE model in Iacoviello (2005):

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{14} \end{bmatrix} [a_t] + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix} \quad (18)$$

where $v_t = Dv_{t-1} + \xi_t$ and $\xi'_t = [\xi_{1,t}, \xi_{2,t}, \xi_{3,t}, \xi_{4,t}]$ is $IN(0, V)$ and uncorrelated with ε_t which is the error term in the AR(1) process of total factor productivity a_t . The trend should not affect inflation and the nominal interest rate since they are assumed to be stationary. If a_t is a unit root process, both output and housing prices should be equally affected by the TFP stochastic trend, since output affects housing prices. This is shown in the log-linear housing demand equation below:

$$\hat{q}_t = \gamma_e \hat{q}_{t+1} + (1 - \gamma_e)(\hat{Y}_{t+1} - \hat{X}_{t+1} - \hat{h}_t) - m\beta \hat{r}_t - (1 - m\beta)\Delta \hat{c}_{t+1} - \phi_e(\Delta \hat{h}_t'' - \beta \Delta \hat{h}_{t+1}'') \quad (19)$$

where $\hat{r}_t$ denotes the real interest rate.

This implies that we should have $d_{11} = d_{12}$. Furthermore, we should have that $d_{13} = d_{14} = 0$ since inflation and the nominal interest rate is assumed stationary. This implies rank $r = 3$ as in (17), which yields

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_1 \\ d_1 \\ 0 \\ 0 \end{bmatrix} [a_t] + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix} \quad (20)$$

The MA representation implies the three stationary relations outlined in (15) and it is also implied that the cumulated shocks to output is the common driving trend of the model.

When the inflation and the nominal interest rate are assumed to be non-stationary, and we have one stationary relationship between all of the four variables as in (16), we have three common driving trends. This implies that not only total factor productivity drives the model, but also two other factors. The theoretical model includes four exogenous shocks, related to productivity, prices (cost-push shock), housing preferences and a shock process for the central bank's interest rate rule. The model in Iacoviello (2005) assumes the shock to the nominal interest rate to be of mean zero, while the three remaining shocks are expressed as stochastic AR(1) processes. If we assume that these may contain a unit root, the MA representation can be expressed as

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \\ d_{14} & d_{24} & d_{34} \end{bmatrix} \begin{bmatrix} \sum u_{1,i} \\ \sum u_{2,i} \\ \sum u_{3,i} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix} \quad (21)$$

where the first stochastic trend is assumed to be the shocks to total factor productivity $\sum u_{1,i} = a_t$. For $q_t - y_t$ to be stationary, we need to have $d_{11} = d_{12}$, $d_{21} = d_{22}$ and $d_{31} = d_{32}$ such that the stochastic trends feeds into y_t and q_t equally. Imposing the restriction for the Fisher relationship (Fisher, 1930) to hold, the stochastic trends should feed equally into π_t and r_t , such that we need $d_{13} = d_{14}$, $d_{23} = d_{24}$ and $d_{33} = d_{34}$.

The third interpretation of the theoretical model is given by (17). This suggests two common driving trends and the following MA representation:

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{21} \\ d_{12} & d_{22} \\ d_{13} & d_{23} \\ d_{14} & d_{24} \end{bmatrix} \begin{bmatrix} \sum u_{1,i} \\ \sum u_{2,i} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix}, \quad (22)$$

where $d_{11} = d_{12}$, $d_{21} = d_{22}$, $d_{13} = d_{14}$ and $d_{23} = d_{24}$ in order for long-run homogeneity between output and house prices and the Fisher relationship to hold. Shocks to TFP and inflation may be the two driving trends of this model.

3 Statistical model

3.1 The Data

In order to make this paper closely linked to the model in Iacoviello (2005), I will use the same data set over the same sample (1974Q1 to 2003Q2) as he does. The only exception is that I instead of using real house prices and real GDP filtered via a band-pass filter removing frequencies above 32 quarters, will use the log of the variables without filtering. I will also not multiply inflation and the interest rate by 100. The variables (except the interest rate) are seasonally adjusted.

The VAR model estimated in Iacoviello (2005) also includes one lag of the log of the Commodity Research Bureau (CRB) price index, a constant, a time trend, and a shift dummy that is zero until 1979Q4 and unity thereafter. I will also include these in the estimated CVAR model. The results were obtained using CATS in RATS, version 2 (Dennis et al., 2006).

3.2 Specification of the VAR model

The common trends (or MA) representation e.g. in (18) was specified for a VAR(1) model. However, two lags are needed in order to get an estimated model without autocorrelation (tests will be shown below). The VAR(2) model may then be expressed in error correction form (ECM) as² (see e.g. Dennis et al. (2006))

$$\Delta x_t = \omega \Delta W_t + (\Gamma_{11} - \omega \Gamma_{21}) \Delta z_{t-1} + \alpha_1 [\beta', \beta_0, \beta_1] \tilde{z}_{t-1} + \Phi D_t + \gamma_0 + \gamma_1 t + \tilde{\varepsilon}_t \quad (23)$$

where $x'_t = [R_t, \pi_t, q_t, Y_t]$, $\tilde{z}'_{t-1} = [z_{t-1}, 1, t]'$, $z'_t = [x_t, W_t]$, $\Gamma'_1 = [\Gamma_{11}, \Gamma_{21}]$, $\alpha' = [\alpha_1, \alpha_2]$, $\tilde{\varepsilon}_t = \varepsilon_{1t} - \omega \varepsilon_{2t}$, $\varepsilon'_t = [\varepsilon_{1t}, \varepsilon_{2t}]$, $W_t = \ln CRB_{t-1}$ and $\varepsilon_t \sim IN(0, \Omega)$ for $t = 1, \dots, T$ and x_{-1}, x_0 is given. This implies that the log of the CRB spot price index is included as a weakly exogenous variable.

The variables are y_t which is log of real GDP (nominal GDP deflated by the GDP deflator), q_t which is log of real house prices (Conventional Mortgage Home Price Index - CMHPI by Freddie Mac deflated by the GDP deflator), π_t which is log difference of the GDP deflator and R_t which is the Fed Funds rate calculated as the average value in the first month of each quarter. The sample is chosen to be similar to the sample used in the estimation in Iacoviello (2005), i.e. 1974Q1 to 2003Q2. D_t is a vector of dummy variables, μ_0 is a constant and μ_1 is a coefficient for the trend t .

In order to prevent quadratic trends in (23), the trend component needs to be restricted to the cointegration relations such that $\beta_1 \neq 0$ and $\gamma_1 = 0$. This enables the cointegration space to include time as a trend-stationary variable such that unknown exogenous growth may be accounted for (e.g. technical progress) (Harris and Sollis, 2003, p. 133). The constant term needs to be unrestricted in order to allow for a constant term in the equations describing the slope of the linear trends in the data (Juselius, 2006). Including the trend is in line with the estimated VAR in Iacoviello (2005), although the theoretical model as outlined in the MA representation in e.g. (18) suggests that the trend should not be included in the cointegrating relations since the trend of y_t and q_t should cancel. We may therefore expect to be able to exclude the trend after estimating the unrestricted model.

3.3 Institutional events

If we consider the extraordinary institutional events to be given as non-normal residuals, it is shown that the VAR variables can not explain the event. By using no variables representing these events (which may be results of interventions, reforms, etc.) except the shift dummy for 1979Q4 and thereafter, the model is misspecified. Extending the model to include 3 lags does not improve this.

²Note the change of ordering of the variables in x_t in the estimated model compared to the ordering in the MA representation of the theoretical model. This is done to use similar ordering as in the estimated VAR model in Iacoviello (2005).

Multivariate tests (p-values underneath test values)	
Residual autocorrelation LM(1)	$\chi^2(16) = 33.315$ (0.007)
Residual autocorrelation LM(2)	$\chi^2(16) = 55.280$ (0.000)
Test for normality	$\chi^2(8) = 177.267$ (0.000)
Test for ARCH LM(1)	$\chi^2(100) = 257.700$ (0.000)

Table 1: Misspecification tests for the full period

In order to get a well-specified model which includes no autocorrelation and normality we need to add dummy variables for as many as 19 periods according to Autometrics' (Doornik, 2006) large outlier detection, which should account for institutional events. Although the model contains autoregressional conditional heteroskedasticity (ARCH), the VAR results should be robust to moderate ARCH effects (Rahbek et al., 2002). Adding dummy variables for the periods which have large outliers yields the results in table 2 (where "Dp" indicates a permanent innovation and "Dtr" a transitory innovation for the mentioned and the preceding period) and table 3.

Dummy variable	F(4,83)	p-value	Dummy variable	F(4,83)	p-value
Dp1974Q3	16.86	0.00	Dp1980Q3	16.05	0.00
Dp1975Q1	2.14	0.00	Dp1980Q4	11.37	0.00
Dp1975Q2	1.57	0.19	Dp1981Q1	30.86	0.00
Dtr1975Q3	6.79	0.00	Dp1981Q3	13.50	0.00
Dp1976Q1	4.97	0.00	Dp1982Q2	10.87	0.00
Dp1976Q3	3.59	0.01	Dp1982Q3	3.27	0.02
Dp1977Q2	8.55	0.00	Dp1983Q1	1.83	0.13
Dp1977Q4	4.93	0.00	Dp2000Q1	1.73	0.15
Dp1978Q2	10.10	0.00	Dp2000Q2	2.18	0.08
Dp1980Q2	21.85	0.00			

Table 2: F-tests on the dummy variables

Multivariate tests (p-values underneath test values)	
Residual autocorrelation AR 1-5	$F(80, 250) = 1.278$ (0.080)
Test for normality	$\chi^2(8) = 5.536$ (0.699)
Test for ARCH	$F(84, 290) = 1.523$ (0.006)

Table 3: Misspecification tests for the full period after adding dummy variables

We see from table 3 that the estimated VAR is now quite well-specified, but we would like not to add as many dummy variables as 19 to the model. We also see that the vast majority of the dummy variables belong to the first part of the sample.

Following Juselius and Franchi (2007), I split the sample in one period going until 1979Q4 (prior to the second oil crisis) and a second period going from 1981Q2 until the end of the

sample. In this way, the period 1980Q1-1981Q1 is dropped in order to avoid the outlier observations corresponding to these periods. Iacoviello (2005) splits the sample in his VAR estimation with a shift dummy in 1979Q4 such that the sample split done here is in line with what Iacoviello (2005) does. However, since the sample split period starting from the beginning of the sample (1974Q1) going until 1979Q4 is quite short (24 observations), I choose to only do the estimation for the second split of the sample. Hence, the sample in the estimated CVAR model in this paper is 1981Q2-2003Q2. In order to correct for large outliers pertaining to the residuals for the nominal interest rate, we need to impose dummies for the periods 1981Q4 and 1982Q2 and no other periods which is in line with the Volker disinflation policy period. This gives a well-specified model with normality, no autocorrelation and no ARCH.

Multivariate tests (p-values underneath test values)	
Residual autocorrelation LM(1)	$\chi^2(16) = 24.746$ (0.074)
Residual autocorrelation LM(2)	$\chi^2(16) = 34.466$ (0.005)
Test for normality	$\chi^2(8) = 12.691$ (0.123)
Test for ARCH LM(1)	$\chi^2(100) = 85.430$ (0.850)

Table 4: Misspecification tests for the split sample

3.4 Exclusion

The first hypothesis related to the theoretical model that we would like to test is whether the trend is excludable from the long run relations. If it is found to be excludable, the model may be estimated without a trend since the trend will not provide any useful information to the long-run structure (Juselius, 2006, p. 351). Since all the linear trends in the levels should cancel, the trend should be excludable from the long-run relations.

r	DGF	5% C.V.	R	π	q	y	ln(CRB)	TREND
1	1	3.841	26.755 [0.000]	27.637 [0.000]	13.920 [0.000]	0.013 [0.910]	5.079 [0.024]	0.814 [0.367]
2	2	5.991	32.182 [0.000]	48.120 [0.000]	17.215 [0.000]	23.684 [0.000]	8.198 [0.017]	22.929 [0.000]
3	3	7.815	45.489 [0.000]	65.439 [0.000]	17.226 [0.000]	35.986 [0.000]	9.985 [0.019]	29.744 [0.000]

Table 5: Test of long-run exclusion from the VAR for various choices of the rank

The results in table 5 (p-values in brackets below test values) show that the trend and output may be excludable for $r = 1$ while it is significant for $r = 2$ and $r = 3$. This implies that the trend and output is not needed in the first cointegrating relation but is needed for the other cointegrating relations, which is in line with the indications from the rank test after estimating the model without a trend shown in table 8, suggesting that the output and the linear trend constitutes the second cointegrating relationship. Also, if a variable is found to be excludable according to this test, it may be a sign of strong multicollinearity with other variables and it may suggest that some relevant variables are missing from the cointegration analysis (Dennis et al., 2006, p. 72). The estimated model without a trend shows, according

to the exclusion test shown in table 6, that output may not be excluded from the long-run relationships. This suggests that there is strong multicollinearity between the linear trend and the log of real output. According to the theoretical long-run relations, output should be included and the trend should be excluded, so I choose to keep output when estimating the long-run relations but restricting the trend to be excluded.

r	DGF	5% C.V.	R	π	q	y	$\ln(CRB)$
1	1	3.841	32.288 [0.000]	46.738 [0.000]	15.516 [0.000]	11.491 [0.001]	4.357 [0.037]
2	2	5.991	40.610 [0.000]	54.614 [0.000]	15.938 [0.000]	15.117 [0.001]	7.756 [0.021]
3	3	7.815	48.573 [0.000]	67.169 [0.000]	19.248 [0.000]	20.864 [0.000]	10.399 [0.015]

Table 6: Test of long-run exclusion in model without trend for various choices of the rank

3.5 Rank

The theoretical models in (15), (16) and (17) correspond to different choices of the rank ($r = 3$, $r = 1$ and $r = 2$). I will use statistical tests in order to determine the rank together with these theoretical interpretations. This will pick out one of the three models pertaining to the three different rank options, which will be used as the relevant model for the further analysis in the paper.

r	$p - r$	λ_i	p	p^*	ρ_{max}
0	4	0.565	0.000	0.000	0.603
1	3	0.402	0.000	0.003	0.620
2	2	0.215	0.239	0.568	0.891
3	1	0.036	0.947	0.960	0.861

Table 7: Determining the rank based on the unrestricted VAR model

The trace test suggests a rank of $r = 2$ (the row marked $r = i$ tests $H_0 : r \leq i$) for a 5% significant level, both using the Bartlett corrected test (the column marked p^* in table 7 shows the p-value) and the standard trace test (the column marked p in table 7 shows the p-value). The (modulus of) the largest unrestricted characteristic root for all choices of the rank suggests a rank of $r = 1$. Setting $r = 3$ would leave a root of 0.861 in the model (see the column marked ρ_{max} in table 7), which is quite large and choosing $r = 2$ would give the largest root equal to 0.891, and if rank $r = 1$ the modulus of the largest unrestricted root is as small as 0.62. The largest unrestricted root should be significantly different from unity in order to do the various stationarity tests conducted later in the paper, but since we do not know the standard deviation of the roots, we do not know exactly whether the reported roots suggest a rank of $r = 1$. Still, there is a quite small increase in the largest root when going from rank $r = 3$ to $r = 2$ such that we should set the rank to $r = 1$ if looking at the roots. The column marked λ_i shows the estimated eigenvalues, which should be low.

The theoretical model suggest a rank of $r = 3$ for the scenario in (15), rank $r = 2$ for (17) and rank $r = 1$ for the scenario in (16). This implies that the choice of rank based on the statistical tests will influence which theoretical scenario we will pursue in the analysis that follows.

3.5.1 Estimating the model without the trend

Since the theoretical implication suggests that there should be no deterministic trend in the model, we should try to test for rank using an estimated model where we do not include the trend. The estimated cointegrated VAR model below will also show that the trend may be excludable in the long-run relationships, which also indicates that an estimated model without a trend could be a good representation of the data. However, the estimated VAR model in Iacoviello (2005) includes a trend, so I will include the trend in the estimated model in the analysis below. In order to do a sensitivity analysis regarding finding the rank I will perform a rank test where the trend is not included in the model. The trace test and the modulus of the largest unrestricted roots then yields

r	$p - r$	λ_i	p	p^*	ρ_{max}
0	4	0.561	0.000	0.000	0.603
1	3	0.229	0.392	0.612	0.616
2	2	0.151	0.798	0.923	0.856
3	1	0.014	0.999	0.999	0.901

Table 8: Determining the rank in model with no trend

The model estimated without a trend suggests a rank of $r = 1$ both when looking at the trace test and the modulus of the largest unrestricted root. This may be a sign that the second cointegrating relationship found by the trace test for the model which included a trend were a relationship between the trend and output yielding a trend-stationary output gap. The tests of exclusion shown in table 5 and table 6 also supports this proposition.

3.6 Stationarity

r	DGF	5% C.V.	R	π	q	y
1	3	7.815	48.470 [0.000]	41.265 [0.000]	62.473 [0.000]	31.118 [0.000]
2	2	5.991	24.099 [0.000]	24.879 [0.000]	35.281 [0.000]	3.426 [0.180]
3	1	3.841	0.452 [0.502]	1.318 [0.251]	13.774 [0.000]	1.556 [0.212]

Table 9: Test of stationarity

r	DGF	5% C.V.	R	π	q	y
1	3	9.488	59.271 [0.000]	52.740 [0.000]	62.648 [0.000]	59.950 [0.000]
2	2	7.815	32.699 [0.000]	31.792 [0.000]	35.331 [0.000]	32.363 [0.000]
3	1	5.991	12.545 [0.002]	11.487 [0.003]	19.386 [0.000]	20.222 [0.000]

Table 10: Test of stationarity in model with excluded trend

The stationarity test in table 9 is a test for trend-stationarity since the trend is included in the cointegrating relations, while the test in table 10 is for stationarity where the trend is

excluded from the cointegrating relations, conditional on the choice of rank. Since none of the variables is shown to be stationary, the assumption of stationary inflation and nominal interest rate for rank $r = 3$ according to (15) does not seem to hold. The test for trend-stationarity shows that output, the nominal interest rate and inflation are trend-stationary when rank is $r = 3$. Since housing prices are not trend-stationary while the other variables are, different trends may drive the financial markets and the real economy. This is evidence against long-run homogeneity between housing prices and output.

4 Testing the long-run relations

The test and choice of rank above determined the number of unique cointegration vectors spanning the cointegration space. However, any linear combination of the stationary vectors is a stationary vector, such that the estimates for any of the columns in the β matrix are not unique. It is therefore necessary to impose restrictions in order to test if the columns of β are identified (Harris and Sollis, 2003, p. 143). Since the rank test suggested a rank of $r = 1$, I will use the theoretical scenario pertaining to (16) and disregard the other scenarios. This implies one stationary relationship and three common driving trends in the model. The other scenarios (from (15) and (17)) are also tested, and the results are shown in table 12 and appendix D.

For rank $r = 1$ in (16), we only have one cointegration vector, which should include the four endogenous variables, but not the exogenous variable log of the CRB price index or the trend. There should also be long-run homogeneity between housing prices and output and between the nominal interest rate and inflation (corresponding to the definition of the real interest rate according to the Fisher relationship). Imposing these restrictions jointly yields the results given in the first row in table 12.

	R	π	q	y	$\ln(crb)$	Trend
β_1	29.30 (2.93)	-29.30 (-2.93)	1	-1	-	-
Test of restricted model: $\chi^2(4) = 56.887[0.000]$						
Test of restricted model, Bartlett corrected: $\chi^2(4) = 44.896[0.000]$						

Table 11: Testing imposed theoretical restrictions

The model after imposing rank $r = 1$ and the restrictions pertaining to the hypotheses in (16) yields the long-run relationship from $\beta'x_{t-1}$

$$\ln q_t = \ln Y_t - 29.30(R_t - \pi_t), \quad (24)$$

which expresses an inverse demand function for housing that is the only stationary relation among the variables. However, the hypotheses are not accepted when imposed on the cointegration vector of the estimated model. This is shown in table 11 and figure 1, where we observe a non-stationary relationship. The upper panel of figure 1 shows the residuals from the estimated long-run cointegrating relation and the lower panel of figure 1 shows the residuals from the estimated long-run cointegrating relation corrected for short-run effects.

Theoretical scenario	$\chi^2_{.95}(\nu)$	p-value	$\chi^2_{.95}(\nu)$ Bartl.	p-value Bartl.
$r = 1$	56.887(4)	0.000	44.896(4)	0.000
$r = 2$	95.007(8)	0.000	9.037(8)	0.339
$r = 3$	63.891(9)	0.000	4.480(9)	0.877

Table 12: Test of long-run relations

The only accepted hypothesis is excluding the trend from the long-run relationship, which also corresponds to the results from the general test for exclusion above.

The long-run relationship from $\beta'x_{t-1}$ is then given as (with absolute values of t-ratios in parentheses below the coefficients)

$$\ln q_t = 0.645 \ln Y_t - 15.828 R_t + 57.300\pi_t + 0.219 \ln CRB_{t-1}, \quad (25)$$

(11.021) (7.657) (10.951) (2.656)

which also shows an inverse demand function for housing, but not with long-run homogeneity between housing prices and output and the Fisher relationship. Both the GDP deflator and log of the CRB price index is significant and included as price measures. This restriction is accepted, with the test values and p-values in brackets $\chi^2(1) = 0.81[0.37]$ and for the Bartlett corrected case $\chi^2(1) = 0.62[0.43]$. This suggests that there may exist a long-run relation pertaining to housing demand, but long-run homogeneity between house prices and output and between the nominal interest rate and inflation does not exist. The cointegrating relation is shown in figure 2, where there seems to be a stationary relationship.

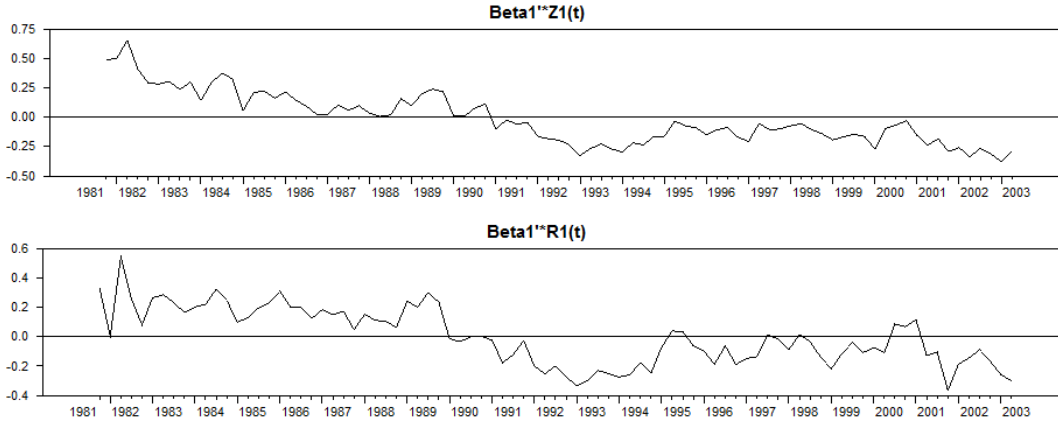


Figure 1: Cointegrating relationship, with restrictions on β

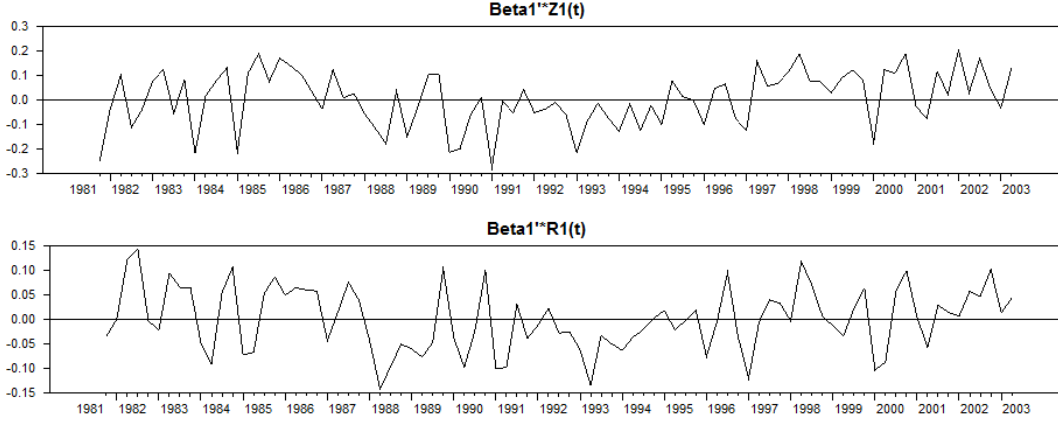


Figure 2: Cointegrating relationship, with excluded trend

5 The moving average (MA) representation

The vector error correction model shown in (23) may also be represented by a moving average structure as shown in Dennis et al. (2006) and Juselius (2006). This will then be given by

$$x_t = C \sum_{i=1}^t (\varepsilon_i + \Psi D_i) + C^*(L)(\varepsilon_t + \mu_0 + \alpha\beta_1 t + \Phi D_t) + A \quad (26)$$

where

$$C = \beta_{\perp}(\alpha'_{\perp}(I - \Gamma_1 - \Gamma_2)\beta_{\perp})^{-1}\alpha'_{\perp} = \tilde{\beta}_{\perp}\alpha'_{\perp} \quad (27)$$

where the values of the parameters in the matrices $\beta_{\perp}$ and $\alpha_{\perp}$ can be calculated from the estimates of α , β and Γ . The matrix A depends on the initial values and satisfies $\beta' A = 0$. This moving average representation may be used to analyze the structure of the common stochastic trends, which represents the pushing forces that create the non-stationary properties of the data. The unrestricted or restricted estimates of α and β can therefore be used in order to find the common stochastic trends and their weights. The cointegration analysis will be covered through restrictions on the β vectors, while the estimate of C and $\alpha_{\perp}$ will give information of the estimated common trends. Since the rank is set to $r = 1$, we have $p - r = 3$ common stochastic trends.

The estimated CVAR model with the theoretical restrictions given by (16) imposed, may be transformed into a common trends representation using the estimated parameters pertaining to the matrices α , β and Γ . This yields the results for the parameters in (26), given in table 13 which shows the matrices which is relevant for the common trends representation with t-values in braces below the parameters.

	$\hat{\varepsilon}_R$	$\hat{\varepsilon}_\pi$	$\hat{\varepsilon}_q$	$\hat{\varepsilon}_y$
$\hat{\sigma}_{\varepsilon_i}$	0.0017	0.0016	0.0122	0.0108
$\hat{\alpha}_{\perp,1}^c$	0.23 [0.54]	1.00	0.00	0.00
$\hat{\alpha}_{\perp,2}^c$	-0.04 [-0.04]	0.00	0.00	1.00
$\hat{\alpha}_{\perp,3}^c$	-1.61 [-1.32]	0.00	1.00	0.00
	R	π	q	y
$\hat{\beta}_{\perp,1}^c$	4.25 [0.77]	0.80 [4.78]	1.83 [1.39]	1.01 [0.86]
$\hat{\beta}_{\perp,2}^c$	0.22 [3.14]	0.13 [1.97]	-0.69 [-1.35]	2.03 [4.49]
$\hat{\beta}_{\perp,3}^c$	0.05 [0.73]	0.10 [1.66]	2.13 [4.33]	0.55 [1.27]
The C matrix				
	$\hat{\varepsilon}_R$	$\hat{\varepsilon}_\pi$	$\hat{\varepsilon}_q$	$\hat{\varepsilon}_y$
R	0.09 [0.33]	0.77 [4.25]	0.05 [0.74]	0.22 [3.14]
π	0.02 [0.06]	0.80 [4.78]	0.10 [1.66]	0.13 [1.97]
q	-2.97 [-1.47]	1.83 [1.39]	2.13 [4.33]	-0.69 [-1.35]
y	-0.74 [-0.41]	1.01 [0.86]	0.55 [1.27]	2.03 [4.49]

Table 13: Parameters for the MA representation

Using the estimated values, we get the following estimated common trends representation:

$$\begin{bmatrix} R_t \\ \pi_t \\ q_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.77 & 0.22 & 0.05 \\ [4.25] & [3.14] & [0.73] \\ 0.80 & 0.13 & 0.10 \\ [4.78] & [1.97] & [1.66] \\ 1.83 & -0.69 & 2.13 \\ [1.39] & [-1.35] & [4.33] \\ 1.00 & 2.03 & 0.55 \\ [0.86] & [4.49] & [1.27] \end{bmatrix} \begin{bmatrix} \sum u_{1,i} \\ \sum u_{2,i} \\ \sum u_{3,i} \end{bmatrix} + \text{stationary and det. components} \quad (28)$$

The stochastic trends may be interpreted as (with t-ratios in brackets below the coefficients)

$$\begin{aligned} \sum u_{1,i} &= \sum \varepsilon_\pi + \frac{0.23}{[0.54]} \sum \varepsilon_R \\ \sum u_{2,i} &= \sum \varepsilon_y - \frac{0.04}{[0.03]} \sum \varepsilon_R \\ \sum u_{3,i} &= \sum \varepsilon_q - \frac{1.61}{[1.32]} \sum \varepsilon_R \end{aligned}$$

Disregarding the insignificant variables, we see that the first stochastic trend can be interpreted as cumulated shocks to inflation, the second as cumulated shocks to output and the third as cumulated shocks to housing prices, as the theoretical model may suggest. We may

therefore write the MA representation as

$$\begin{bmatrix} R_t \\ \pi_t \\ q_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.77 & 0.22 & 0.05 \\ [4.25] & [3.14] & [0.73] \\ 0.80 & 0.13 & 0.10 \\ [4.78] & [1.97] & [1.66] \\ 1.83 & -0.69 & 2.13 \\ [1.39] & [-1.35] & [4.33] \\ 1.00 & 2.03 & 0.55 \\ [0.86] & [4.49] & [1.27] \end{bmatrix} \begin{bmatrix} \sum \varepsilon_\pi \\ \sum \varepsilon_y \\ \sum \varepsilon_q \end{bmatrix} + \text{stationary and det. components} \quad (29)$$

The nominal interest rate is mainly influenced by cumulated shocks to output and inflation while inflation is mainly influenced by cumulated shocks to inflation only. This is evidence of non-stationarity of the real interest rate, which rejects the hypothesis pertaining to the second relation in (17). Housing prices are mainly influenced by cumulated shocks to itself, and output is influenced by cumulated shocks to itself. This may indicate evidence against a stationary relationship between output and housing prices, since two separate trends are driving them. We also see that there are few parameter estimates corresponding to the expected relationships from (21) ($d_{11} = d_{12}$, $d_{21} = d_{22}$, $d_{31} = d_{32}$, $d_{13} = d_{14}$, $d_{23} = d_{24}$ and $d_{33} = d_{34}$).

The estimated C-matrix in table 13 suggests that cumulated empirical shocks to the nominal interest rate only have transitory effects to the variables of the system since all the parameters in the first column of the C-matrix are insignificant. It also suggests that cumulated shocks to inflation have permanent effects on the nominal interest rate and inflation, cumulated shocks to housing prices have only permanent effects on housing prices and cumulated shocks to GDP have permanent effects on the nominal interest rate and GDP. The normalization of $\alpha'_\perp$ as shown in table 13 and the interpretation of the three stochastic trends is in line with this.

6 Structural MA model and impulse response functions

We can impose the orthogonality condition on the residuals while separating between r transitory shocks, u^T , and $p - r$ permanent shocks, u^P , and impose exclusion restrictions on the parameters of the model, in order to identify the structural shocks $u_t = (u_t^T, u_t^P)$ (Juselius, 2006, p. 275) and (Dennis et al., 2006). These structural shocks can then be given an economic interpretation in order to extend the analysis of the model and relate it to the theoretical scenarios.

The MA representation is given by (26). We may relate the VAR residuals ε_t to the structural shocks u_t by a matrix B such that

$$u_t = B\varepsilon_t \Leftrightarrow \varepsilon_t = B^{-1}u_t, \quad (30)$$

where the error terms u_t are *iid* $N_p(0, I_p)$ (Dennis et al., 2006). By inserting for ε_t in (26) using matrix B , we get³

³The weakly exogenous variable $\ln CRB_{t-1}$ is excluded here to ease notation.

$$x_t = CB^{-1} \sum_{i=1}^t (u_i) + C \sum_{i=1}^t \Phi D_i + C^*(L)(B^{-1}u_t + \mu_0 + \Phi D_t) + A \quad (31)$$

$$= \tilde{C} \left(\sum_{i=1}^t u_i + \sum_{i=1}^t B\Phi D_i \right) + \sum_{i=0}^{\infty} \tilde{C}_i^* (u_{t-i} + B\Phi D_{t-i}) + A \quad (32)$$

where $\tilde{C} = CB^{-1}$ and $\tilde{C}^*(L) = C^*(L)B^{-1}$. I choose to normalize the transitory shock on the nominal interest rate, the first permanent shock on inflation, the second permanent shock on output and the third permanent shock on housing prices. This is in line with the theoretical model which assumes AR(1) processes for the shocks pertaining to the three latter variables, while shocks to the nominal interest rate is assumed to be white noise. Results from the estimated $\alpha'_\perp$ in the MA representation as shown in table 13 also supports this. Additionally, I restrict the second permanent shock (output) to have no long-run effect on inflation and the third permanent shock (housing prices) to have no long-run effect on inflation and the nominal interest rate as shown in (34). This is in line with the theoretical model that suggests no long-run impact on inflation and the nominal interest rate which is assumed stationary in the long run.

The (normalized) rotation matrix B determines how the orthogonalized permanent and transitory shocks are associated with the estimated VAR residuals, and is given by

$$B = \begin{bmatrix} 1.00 & 0.06 & 0.07 & -0.04 \\ 0.02 & 1.00 & 0.13 & 0.16 \\ 0.84 & -0.49 & -0.62 & 1.00 \\ -1.83 & -0.84 & 1.00 & 0.54 \end{bmatrix} \quad (33)$$

The structural long-run impact matrix is given by $\tilde{C} = CB^{-1}$

$$\tilde{C} = \begin{bmatrix} 0.00 & 1.02 & 0.08 & \mathbf{0.00} \\ 0.00 & 1.00 & \mathbf{0.00} & \mathbf{0.00} \\ 0.00 & 2.36 & -1.33 & 1.00 \\ 0.00 & 2.94 & 1.00 & 1.00 \end{bmatrix}, \quad (34)$$

where the assumed exclusion restricted are in bold face.

The impulse response functions from the structural MA with imposed restrictions on β in line with (16) are shown in figure 3.

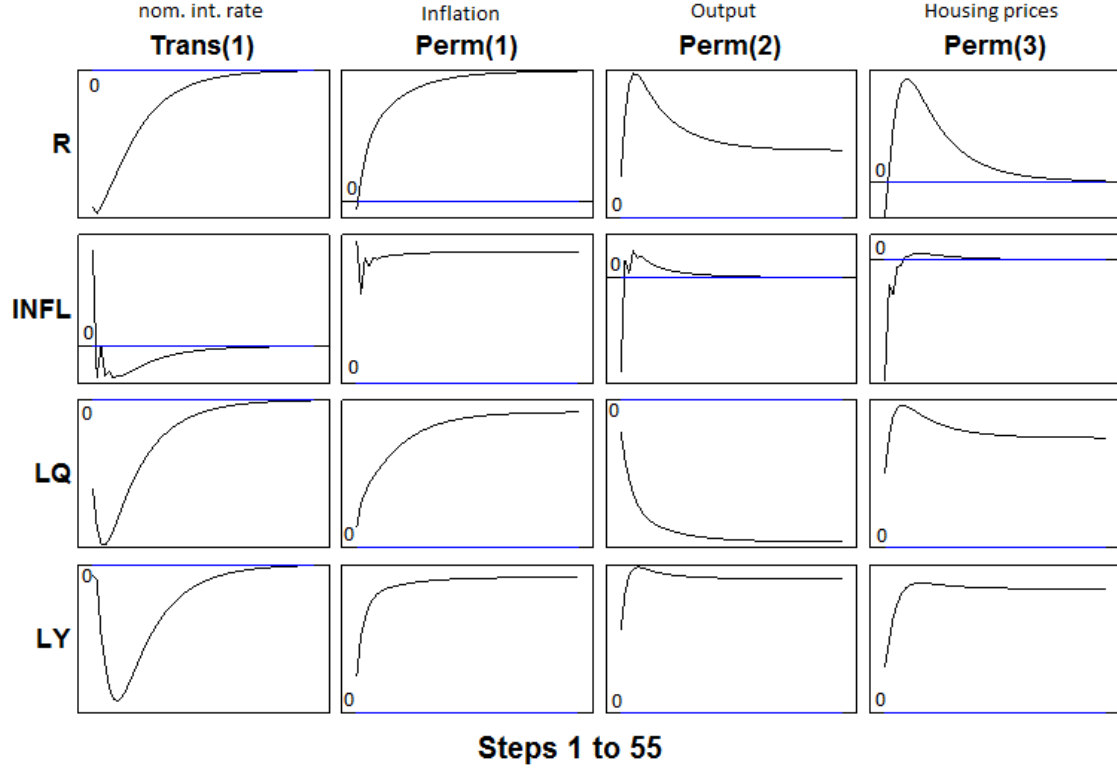


Figure 3: Impulse response functions, structural MA

The impulse response functions given by the restricted CVAR model represented in a structural MA form may be compared to the impulse response functions given by the estimated VAR model in Iacoviello (2005). However, all the variables are assumed non-stationary in the CVAR model such that some shocks need to have permanent effects in order to "drive" these variables. The first column in figure 3 shows the transitory shock, which may be interpreted as a shock to the nominal interest rate. Column two through column three represents the $p - r = 3$ permanent shocks. Ordered from left to right, these may be interpreted as shocks to inflation, output and housing prices respectively. The residuals are restricted to be orthogonal. The impulse response functions given by the estimated VAR model in Iacoviello (2005) are given in figure 4. Each column in figure 3 represents a shock, while each row in figure 4 is related to a shock. We should therefore compare the columns in figure 3 to the rows in figure 4.

6.1 Analyzing the impulse response functions

In order to separate between the impulse response functions in Iacoviello (2005) and the impulse response functions from the structural MA model following the restricted cointegrated VAR model estimated here, I will calculate two additional sets of impulse response functions. The differences in the impulse response functions that originate from using filtered variables instead of unfiltered variables may be channeled by using the same method as in Iacoviello (2005) in order to calculate IRFs, but replacing the filtered variables used in the estimated VAR of Iacoviello (2005) by the unfiltered variables used in the CVAR model estimated here

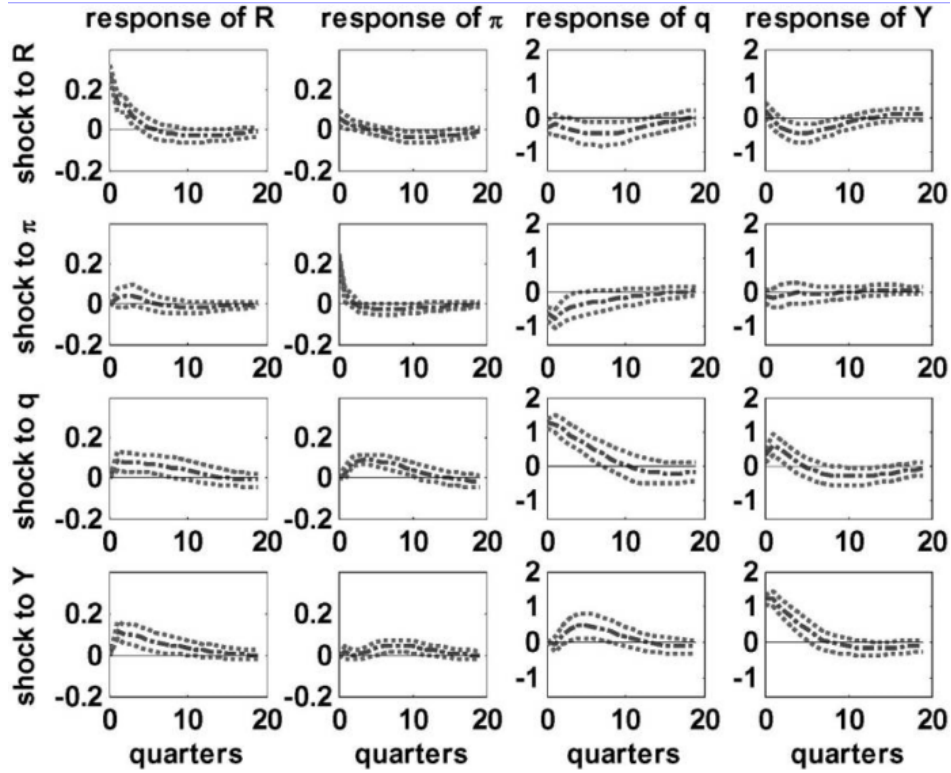


Figure 4: Impulse responses from the estimated VAR model in Iacoviello (2005)

as shown in figure 5. Additionally, I will calculate the IRFs from the structural MA model following the CVAR model as outlined in the preceding chapters of this paper, but using filtered variables to estimate the CVAR model as shown in figure 6 which also helps identify the effects of using unfiltered rather than filtered data. I also show the IRFs following the structural MA model without imposing long-run restrictions on β in 7, in order to see if imposing long-run homogeneity between housing prices and output and imposing the Fisher equation causes differences in the results.

We see from figure 5 (although the scalings on the axes are changed) that the estimated VAR in Iacoviello (2005) using unfiltered variables provides the same results as when using filtered variables except for the shocks being more persistent in the case of unfiltered variables.

The transitory shock may be compared to the shock to R in Iacoviello (2005) as shown in figure 4, except for the direction of the shock being opposite. If we interpret this monetary policy shock as in Iacoviello (2005), where the interest rate increases, the results in Iacoviello (2005) indicate that inflation and housing prices are slightly increased, and output is increased followed by a reduction. The structural MA model with restrictions would, given a positive shock to the nominal interest rate, lead to decreased inflation and temporary increases in housing prices and output. The effect on housing prices and output are opposite of results from the unrestricted VAR of Iacoviello (2005). Using unfiltered variables seems to affect this, looking at 6, but from 7 we see that imposing restrictions on the long-run relations also affects our results.

The first permanent shock in the structural MA model can be interpreted as shocks to inflation. We see that there is a transitory increase in the interest rate. Housing prices and

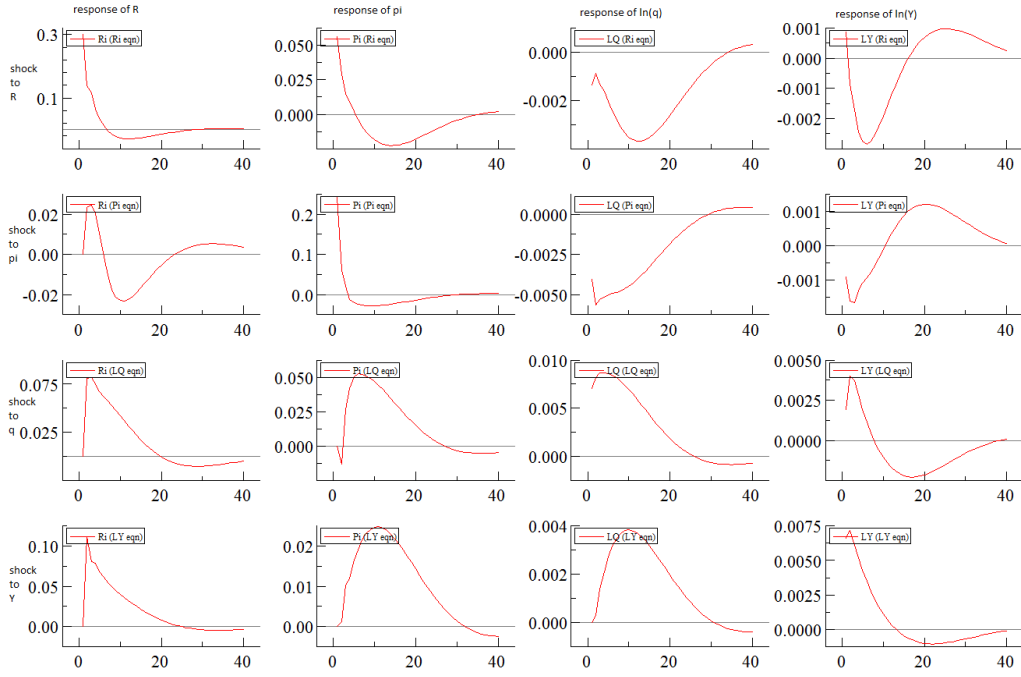


Figure 5: Impulse responses from the estimated VAR model in Iacoviello (2005) using unfiltered variables

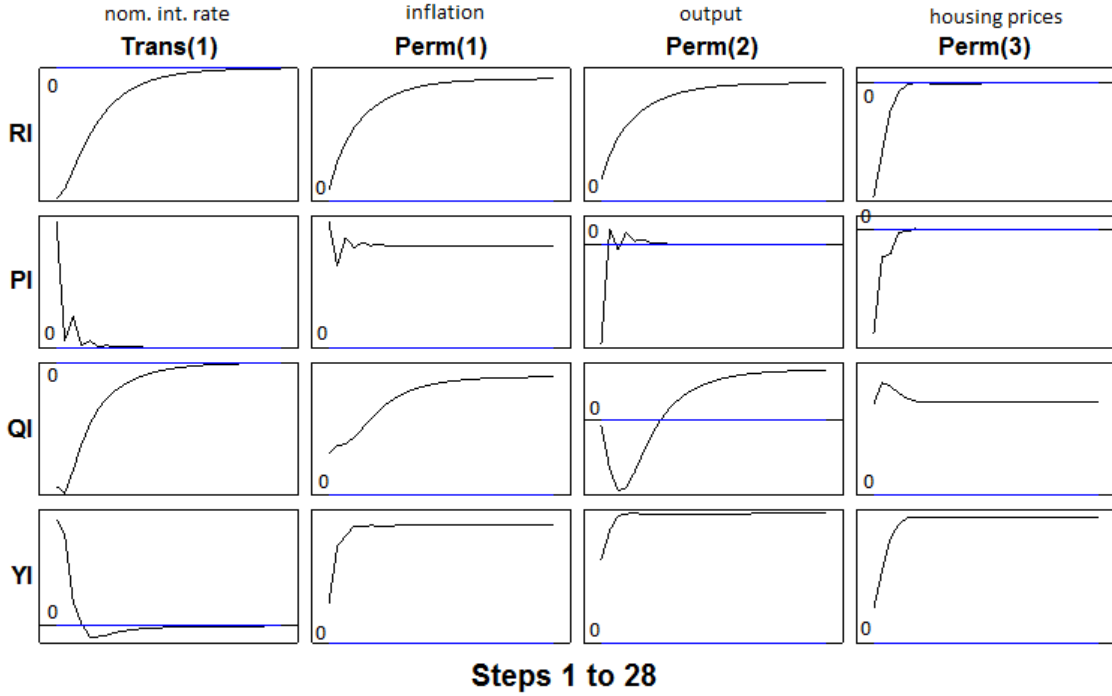


Figure 6: Impulse responses from the estimated structural MA model using filtered variables

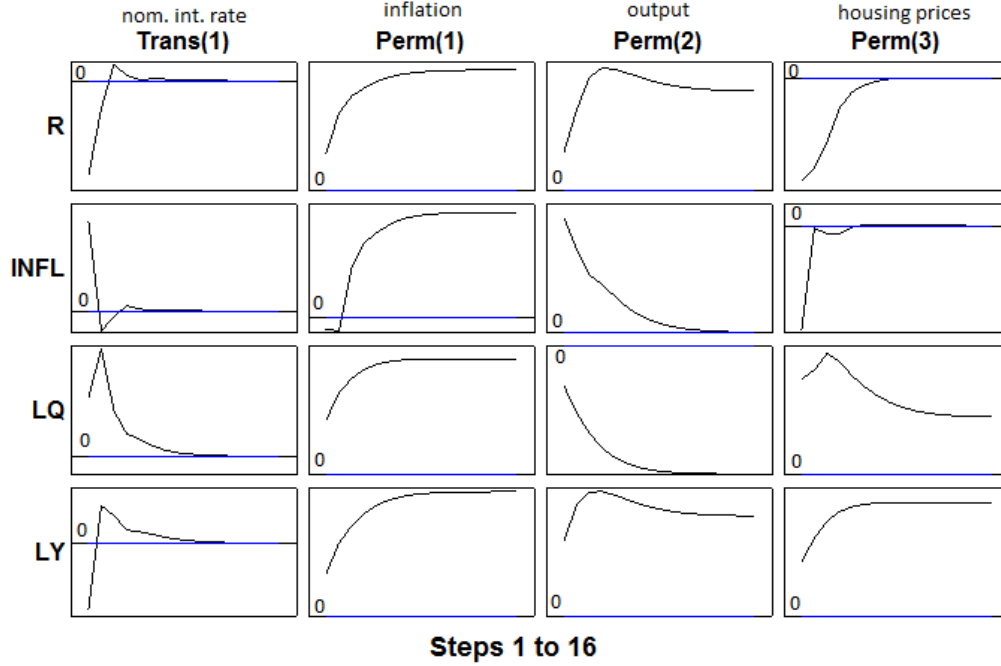


Figure 7: Impulse responses from the estimated structural MA model using unfiltered variables and imposing no long-run restrictions

output increases permanently. A cost-push shock in the unrestricted VAR model also causes increased interest rate, except for the effect on the inflation and the nominal interest rate being permanent in the structural MA model since these variables now are treated as non-stationary. The effect on housing prices and output however, is negative following a cost-push shock in the unrestricted VAR while it is positive in the structural MA model. Since housing prices reacts negatively to the real interest rate, the real interest rate has to decrease in order for the housing prices to increase. This may be the case if the nominal interest rate increase more than inflation, which the imposed fisher relationship could cause. This may explain the difference between the unrestricted VAR and the structural MA model. The structural MA using filtered variables shows the same effects of a cost-push shock, such that the restrictions imposed on β may be the cause of this difference.

The second permanent shock can be interpreted as shocks to output. We see that it is modeled as a positive shock to output, as the shock in the unrestricted VAR. The positive demand shock in the unrestricted VAR causes increased housing prices, interest rate, and inflation. If we consider a positive demand shock in the structural MA model, we see from figure 3 that this causes decreased housing prices, increased nominal interest rate and a transitory decrease in inflation. The imposed long-run homogeneity between housing prices and output should cause a positive demand shock to increase housing prices, but the opposite happens. Since the nominal interest rate increases permanently and inflation decreases temporarily, the real interest rate will increase permanently. This effect needs to be stronger than the demand shock in order for the housing prices to decrease. The effect pertaining to the financial accelerator which should cause increased housing prices following a positive demand shock is therefore not found in the restricted structural MA model. The shocks to output in the

structural MA model with filtered variables show that housing prices are decreased the first periods and then permanently increased. This is in line with the results from the unrestricted VAR (both using filtered and unfiltered variables) in the long run, but not the immediate response. Both the use of unfiltered variables and the identification of the structural MA model therefore seems to change our results, since using unrestricted long-run relations (as shown in figure 7) cause housing prices to increase following a positive demand shock.

Increased housing prices (the third permanent shock), causes a permanent increase in output, a transitory increase in the nominal interest rate and a transitory decrease in inflation in the structural MA model as shown in figure 3. This is evidence of the financial accelerator since increased housing prices causes a permanent increase in output. A positive shock to housing prices causes increased inflation following IRFs from the unrestricted VAR (both using filtered and unfiltered variables), while the IRFs from the structural MA models (all versions of it) shows that inflation decrease following increased housing prices. This therefore seems to be caused by identifying the structural MA model.

The results from the impulse response functions following the structural MA model may often be hard to explain, so the interpretations here are not necessarily correct. Especially, since the imposed long-run restrictions are not accepted when imposing them on the CVAR model, the 'economic labels' imposed on the shocks here may not be correctly connected to the empirical shocks (Juselius, 2006, p. 286). Imposing that the third permanent shock should have no long-run effect on inflation or the nominal interest is also perhaps a strong restriction, since the importance of housing prices which is the label for this shock is then reduced. The housing prices should be an important driving force connecting the financial markets and the real economy. We also see that using filtered variables and imposing restrictions on the long-run relations affects the IRFs such that not only identification of the structural MA model causes other effects than in the unrestricted VAR in Iacoviello (2005).

7 Conclusion

The framework applied in this paper combines the theoretical model and the estimated VAR in Iacoviello (2005) by imposing restrictions on the long-run relations of the theoretical model on the estimated VAR by using the cointegrated VAR model. Even though there is found evidence for the financial accelerator in the estimated VAR and the impulse response functions following the minimum-distance estimation of the theoretical model in Iacoviello (2005), the results from impulse responses of the theoretically restricted structural VAR estimated here does not show evidence for this.

Imposing a constant relationship between housing prices and output according to one of the steady states of the theoretical model implies that an increase in housing prices should lead to a similar increase in output and vice versa. Looking at the time series for inflation and output indicates that this hypothesis does not hold since there has been a much larger growth in housing prices than output for a large period of time. The theoretical model also assumes that inflation and the nominal interest rate should separately be stationary, even though the empirical evidence suggests that these are non-stationary for the sample period the model is being estimated over. Stationarity between the nominal interest rate and inflation, pertaining to the Fisher equation, is therefore imposed on the long-run relations here in order to restrict the real interest rate to be stationary. This seems to be non-stationary as well.

The results following the structural MA model suggests some puzzling results which may

not all be taken too seriously. The imposed long-run homogeneity between housing prices and output and the Fisher equation should impact how the variables respond to various shocks, and since this imposed restriction is not accepted, the interpretation of the results becomes less realistic. Additionally, by using the Band-pass filter on output and real housing prices separately, the potential difference in the trends driving these which is not in line with the long-run homogeneity between housing prices and output may be removed. This is also indicated by the rejection of trend-stationarity of housing prices but acceptance for all the other variables.

The financial accelerator (which is one of the key motivations behind the theoretical model) suggests that increased housing prices should increase aggregate demand which again should increase housing prices, and so on. This indicates that an increase in housing prices should have a self-reinforcing effect such that housing prices might increase more than output, assuming that output growth is limited by increased technology and the housing supply lags housing demand. This would eventually lead to a demand surplus in the housing market following the amplifying effects given by increased housing prices. The long-run homogeneity between housing prices and output may therefore not be consistent with the financial accelerator.

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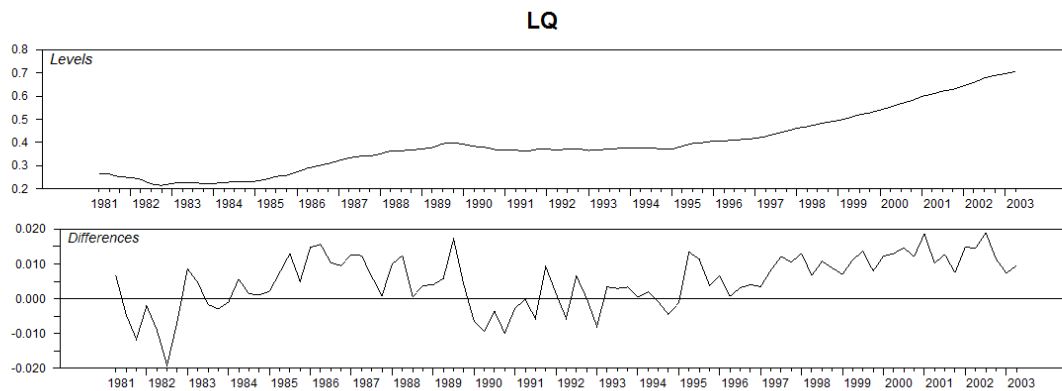
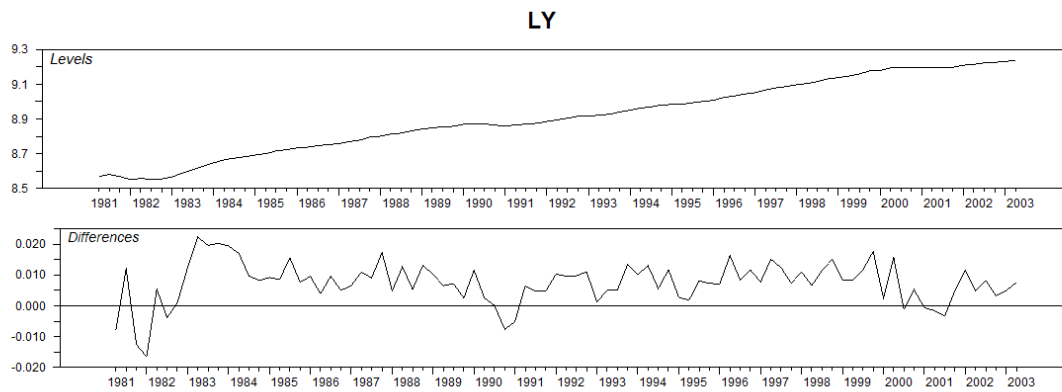
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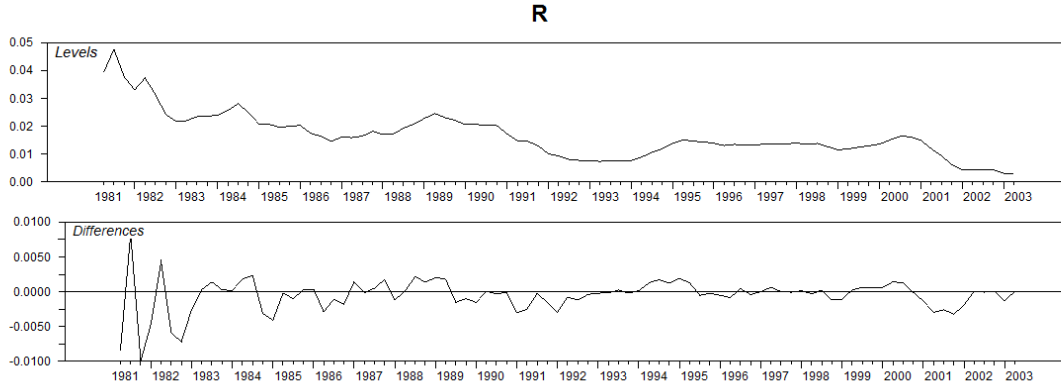
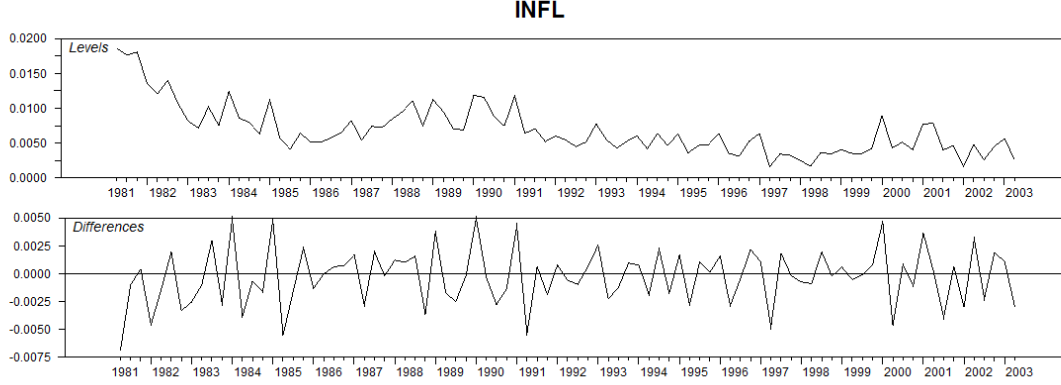
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A The data series





B Testing for univariate integration

In line with the CVAR model, we would like to explain various cointegration relationships. This implies that we search for relationships between variables that are integrated, but where a combination of two or more variables is stationary. The variables that are cointegrated should therefore be integrated, while stationary variables would not need to be linked to a cointegrated variable since it is already stationary. In order to test for cointegration, we need to work with non-stationary time series. We therefore need to perform a univariate unit root test for each variable used in the cointegrated VAR model. This is done below through the augmented Dickey Fuller (ADF) test (Dickey and Fuller, 1981), which estimates the following regression using k lags in x with a constant and a trend:

$$\begin{aligned}
 x_t &= \mu_1 + \mu_2 t + \sum_{i=1}^k A_i x_{t-i} + \varepsilon_t \\
 \Delta x_t &= \mu_1 + \mu_2 t + \pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \varepsilon_t,
 \end{aligned} \tag{35}$$

where we test whether $\pi = 0$. The Akaike information criterion can be used in order to choose the appropriate lag length of the ADF test. Table 14 shows the results for the variables used in the estimated CVAR model.

Table 14: Testing for integration using sample from 1979Q4 to 2003Q2 without any dummy variables

Variable	Null hyp.	Lags	t_{ADF}	Estim. root	$\hat{\sigma}$	AIC
$\Delta^2 RR$	$I(3)$	3	-11.14**	-3.04	0.39%	-11.05
ΔRR	$I(2)$	3	-5.81**	-0.48	0.38%	-11.09
RR	$I(1)$	3	-3.05	0.77	0.37%	-11.16
$\Delta^2 \pi$	$I(3)$	2	-14.17**	-2.51	0.26%	-11.89
$\Delta \pi$	$I(2)$	3	-5.39**	-0.66	0.22%	-12.15
π	$I(1)$	2	-2.33	0.84	0.22%	-12.16
$\Delta^2 LQ$	$I(3)$	1	-12.54**	-0.81	0.64%	-10.08
ΔLQ	$I(2)$	2	-3.40	0.62	0.60%	-10.18
LQ	$I(1)$	3	-1.90	0.96	0.59%	-10.20
$\Delta^2 LY$	$I(3)$	1	-9.70**	-0.68	0.77%	-9.68
ΔLY	$I(2)$	1	-4.73**	0.44	0.71%	-9.87
LY	$I(1)$	2	-3.26	0.89	0.67%	-9.96

As we see from table 14, the nominal interest rate, inflation and log of real GDP are all found to be integrated of order one. Log of real housing prices on the other hand, seems to be integrated of order two. However, the observed t-value for the augmented Dickey-Fuller test on ΔLQ is very close to the 5% critical value of -3.46 , and the alternative hypothesis of the order of integration being smaller than 2 is therefore not accepted. At the 10% significance level the critical value is -3.15 , so if we raise the level of significance slightly above 5% we may accept the alternative hypothesis such that house prices are integrated of order one. This is assumed for the succeeding analysis.

Whether the variables are integrated of order one or zero should be based on statistical arguments such that the statistical inference will produce logically consistent results. In addition, a near-unit root should be approximated with a unit root even though it is significantly different from one (Juselius, 2006, p. 26).

C Derivation of the theoretical long-run relations

This is shown in Iacoviello (2008).

$$\begin{aligned}\frac{q}{Y} &= \zeta_2 \frac{1}{h} \\ \frac{q}{Y} &= \frac{\gamma \nu}{1 - \gamma - (\beta - \gamma)m} \frac{1}{X} \frac{1}{h}\end{aligned}$$

We also have that

$$h = \frac{\zeta_2}{\zeta_3\zeta_7 + \zeta_4\zeta_6 + \zeta_2}$$

Inserting this yields

$$\frac{q}{Y} = \zeta_2 \frac{\zeta_3\zeta_7 + \zeta_4\zeta_6 + \zeta_2}{\zeta_2} = \zeta_3\zeta_7 + \zeta_4\zeta_6 + \zeta_2$$

and the following definitions

$$\begin{aligned}\zeta_2 &= \frac{\gamma\nu}{1 - \gamma - (\beta - \gamma)m} \frac{1}{X} \\ \zeta_3 &= \frac{j'}{1 - \beta} \\ \zeta_4 &= \frac{j''}{1 - \beta'' - m''(\beta - \beta'')} \\ \zeta_6 &= \frac{s''}{1 + (1 - \beta)m''\zeta_4} \\ \zeta_7 &= (s' + (1 - \beta)(m\zeta_2 + m''\zeta_4\zeta_6)) \\ s'' &= (1 - \alpha)(1 - \mu - \nu) \frac{1}{X} \\ s' &= (\alpha(1 - \mu - \nu) + X - 1) \frac{1}{X}\end{aligned}$$

Putting this together gives

$$\begin{aligned}\frac{q}{Y} &= \frac{j'}{1 - \beta} (s' + (1 - \beta)(m\zeta_2 + m''\zeta_4\zeta_6)) \\ &\quad + \frac{j''}{1 - \beta'' - m''(\beta - \beta'')} \frac{s''}{1 + (1 - \beta)m''\zeta_4} + \frac{\gamma\nu}{1 - \gamma - (\beta - \gamma)m} \frac{1}{X} \\ \frac{q}{Y} &= \frac{j'}{1 - \beta} \left(s' + (1 - \beta) \left(m \frac{\gamma\nu}{1 - \gamma - (\beta - \gamma)m} \frac{1}{X} \right. \right. \\ &\quad \left. \left. + m'' \frac{j''}{1 - \beta'' - m''(\beta - \beta'')} \frac{s''}{1 + (1 - \beta)m'' \frac{j''}{1 - \beta'' - m''(\beta - \beta'')}} \right) \right) \\ &\quad + \frac{j''}{1 - \beta'' - m''(\beta - \beta'')} \frac{s''}{1 + (1 - \beta)m'' \frac{j''}{1 - \beta'' - m''(\beta - \beta'')}} + \frac{\gamma\nu}{1 - \gamma - (\beta - \gamma)m} \frac{1}{X}\end{aligned}$$

Taking logs on each side and approximation the right-hand side with a first-order Taylor approximation yields

$$\begin{aligned}
\ln q - \ln Y = & \ln \left(j' \left(s' + \frac{1}{X} m \gamma \frac{\nu}{m\gamma - \gamma + 1} + j'' s'' \frac{m''}{j'' m'' - \beta'' + m'' \beta'' + 1} \right) \right. \\
& + j'' \frac{s''}{j'' m'' - \beta'' + m'' \beta'' + 1} + \frac{1}{X} \gamma \frac{\nu}{m\gamma - \gamma + 1} \Bigg) \\
& + \frac{\beta}{j' \left(s' + \frac{1}{X} m \gamma \frac{\nu}{m\gamma - \gamma + 1} + j'' s'' \frac{m''}{j'' m'' - \beta'' + m'' \beta'' + 1} \right) + j'' \frac{s''}{j'' m'' - \beta'' + m'' \beta'' + 1} + \frac{1}{X} \gamma \frac{\nu}{m\gamma - \gamma + 1}} \\
& \cdot \left(j' \left(s' + j'' s'' m'' \left(m'' \frac{j'' + 1}{(j'' m'' - \beta'' + m'' \beta'' + 1)^2} \right) + \frac{1}{X} m^2 \gamma \frac{\nu}{(m\gamma - \gamma + 1)^2} \right) \right. \\
& \left. \left. + j'' s'' \left(m'' \frac{j'' + 1}{(j'' m'' - \beta'' + m'' \beta'' + 1)^2} \right) + \frac{1}{X} m \gamma \frac{\nu}{(m\gamma - \gamma + 1)^2} \right) \right)
\end{aligned}$$

where $\beta = \frac{\pi}{R}$.

D Long-run relations for other choices of the rank

For rank $r = 3$ in (15), we should exclude all variables except nominal interest rate for the second cointegration vector and all variables except inflation for the third cointegration vector. For the first cointegration vector the interest rate and inflation could be excluded since they are assumed stationary, and there should be homogeneity between housing prices and output. Additionally, the exogenous variable log of the CRB price index and the trend should also be excluded from the cointegration vectors since they are not included in the theoretical scenario. A joint test including all of these restrictions shows that the hypothesis in (15) is rejected when imposed on the cointegrating relationship, but when using the small sample correction of the hypotheses on β derived in Johansen (2002) and Johansen (2000), the restrictions imposed for rank $r = 3$ are accepted, see table 12.

When the rank is $r = 2$, we aim to impose the hypotheses outlined in (17). The joint test shows that the hypothesis related to the scenario for $r = 2$ is rejected, but accepted if we use the Bartlett correction factor outlined in Johansen (2000). The correction factor used in CATS is calculated based on a model without the exogenous variable and institutional dummies (Dennis et al., 2006). This may be the reason why the critical value differs a lot when using the Bartlett correction factor.