Appendix to "Wage bargaining and monopsony" by T. Falch and B. Strøm:

The solution of the model

We assume a 'right to manage' model where the employment is determined by the firm after the wage bargaining. In the wage bargaining, the bargaining parties take into account how the wage will affect the employment in the second stage of the 'game'. If $W > W^*$, the firm set the employment according to the demand $R'(N) = W$, and if $W < W^*$ the firm set employment in accordance with supply $N = \phi(W - A^L)$. Thus, the bargaining game can be seen as maximizing the Nash product $\left\{ \Pi^\gamma (U - U^0)^{(1-\gamma)} \right\}$ with respect to wage and employment *given* these constraints. The problem can then be formulated as maximizing the following Lagrangian with respect to N and W:

$$\ell = \gamma \log \Pi + (1-\gamma) \log \left(N \frac{W - A^L}{2} \right) - \lambda (W - R'(N)) - \mu (N - \phi(W - A^L)), \tag{1}$$

where λ and μ are the Lagrange multipliers associated with demand and supply of labor constraints, respectively. In (1) we use the log of the Nash maximand to simplify the calculations, without altering the results because the maximum is the same. In this maximization problem, at least one of the constraints must be binding, although obviously both constraints do not need to be binding.

The first order conditions and the Kuhn-Tucker conditions are

$$\frac{\partial \ell}{\partial W} = -\gamma \frac{N}{\Pi} + (1-\gamma) \frac{N}{N(W - A^L)} - \lambda + \mu \phi = 0, \tag{2}$$

$$\frac{\partial \ell}{\partial N} = \gamma \frac{R'(N) - W}{\Pi} + (1-\gamma) \frac{W - A^L}{N(W - A^L)} + \lambda R''(N) - \mu = 0, \tag{3}$$

$$\lambda \geq 0 \quad \text{and} \quad \lambda (R'(N) - W) = 0, \tag{4}$$

$$\mu \geq 0 \quad \text{and} \quad \mu (N - \phi(W - A^L)) = 0. \tag{5}$$

Utilizing that $\Pi = R - WN$, (2) and (3) can be written

$$-\gamma N(W - A^L) + (1-\gamma)(R(N) - WN) - (\lambda - \mu\phi)(R(N) - WN)(W - A^L) = 0, \tag{6}$$

$$\gamma (R'(N) - W) + (1-\gamma)(R(N) - WN) + (\lambda R''(N) - \mu) N(R(N) - WN) = 0. \tag{7}$$

In search for global maximum, one has to consider the local maximums in four cases; (i) $\lambda > 0$ and $\mu = 0$, (ii) $\lambda = 0$ and $\mu > 0$, (iii) $\lambda > 0$ and $\mu > 0$, and (iv) $\lambda = \mu = 0$. The last case is not relevant in a 'right to manage model' and will not be discussed here.

Consider first the case (i), i.e., labor demand is binding while labor supply is not, and employment is lower than supply. Inserting into (7) for λ from (6), and utilizing that $R'(N) = W$, one gets

$$
(1-\gamma)\big(R(N)-WN\big)+R''(N)N\left((1-\gamma)\frac{R(N)-WN}{W-A^L}-\gamma N\right)
$$

$$
=(1-\gamma)\left(\frac{1}{\kappa}-1\right)WN+\varepsilon W\left((1-\gamma)\frac{\left(\frac{1}{\kappa}-1\right)WN}{W-A^L}-\gamma N\right)=0,
\tag{8}
$$

where $\kappa = R'(N)\frac{N}{R(N)} > 0$ and $\varepsilon = R''(N)\frac{N}{R'(N)} < 0$. Interior solutions require $\kappa \in \langle 0,1 \rangle$ and $\varepsilon \in \langle -1,0 \rangle$. (8) can be written

$$
(1-\gamma)(1-\kappa)\big(W-A^L\big)+(1-\gamma)(1-\kappa)\varepsilon W-\gamma\kappa\varepsilon\big(W-A^L\big)=0,
\tag{9}
$$

and then the wage is given by

$$
W^D=\frac{(1-\gamma)(1-\kappa)-\gamma\kappa\varepsilon}{(1-\gamma)(1-\kappa)(1+\varepsilon)-\gamma\kappa\varepsilon}A^L=A^L\left(1-\frac{(1-\gamma)(1-\kappa)\varepsilon}{(1-\gamma)(1-\kappa)(1+\varepsilon)-\gamma\kappa\varepsilon}\right)>A^L.
\tag{10}
$$

Obviously, (10) cannot hold if the bargaining strength of the union is very low. In order for (10) to hold, the outcome must be a wage that is such that labor supply is not binding, that is $W > W^*$. The bargaining strength of the firm γ must be sufficiently low. From (10) it follows that this condition is fulfilled only if

$$
\big((1-\gamma)(1-\kappa)-\gamma\kappa\varepsilon\big)A^L > \big((1-\gamma)(1-\kappa)(1+\varepsilon)-\gamma\kappa\varepsilon\big)W^*
$$

$$
\gamma\big((1-\kappa)\big((1+\varepsilon)W^*-A^L\big)+\kappa\varepsilon\big(W^*-A^L\big)\big)>(1-\kappa)\big((1+\varepsilon)W^*-A^L\big)
\tag{11}
$$

$$
\gamma\big((\kappa\varepsilon+1-\kappa)\big(W^*-A^L\big)+(1-\kappa)\varepsilon W^*\big)>(1-\kappa)\big((1+\varepsilon)W^*-A^L\big).
$$

It follows from (10) that $W^D \le A^L/(1+\varepsilon)$ for $\gamma \ge 0$, and strict inequality holds for $\gamma > 0$. Thus

$$
\big((1-\kappa)\big((1+\varepsilon)W^*-A^L\big)+\kappa\varepsilon\big(W^*-A^L\big)\big)<0, \text{ and}
$$

$$
\gamma<\frac{(1-\kappa)\big(A^L-(1+\varepsilon)W^*\big)}{(1-\kappa)\big(A^L-(1+\varepsilon)W^*\big)-\kappa\varepsilon\big(W^*-A^L\big)}\equiv\hat{\gamma}.
\tag{12}
$$

Consider next the case when supply is binding while demand is not, i.e. $\lambda = 0$ and $\mu > 0$. Inserting into (6) for μ from (7), one gets

$$-\gamma N\left(W - A^L\right) + (1-\gamma)\left(R(N) - WN\right) + \phi\left(\gamma\left(R'(N) - W\right) + (1-\gamma)\left(R(N) - WN\right)\right)\frac{W-A}{N} = 0. \quad (13)$$

Utilizing that $\phi = N/\left(W - A^L\right)$, (13) can be written

$$-\gamma\left(W - A^L\right) + (1-\gamma)\left(\frac{R(N)}{N} - W\right) + \gamma\left(\kappa\frac{R(N)}{N} - W\right) + (1-\gamma)\left(\frac{R(N)}{N} - W\right) = 0, \quad (14)$$

and

$$W^S = \frac{1}{2}\gamma A^L + \frac{R(N)}{N}\left((1-\gamma) + \frac{1}{2}\gamma\kappa\right) = \gamma W^M + (1-\gamma)\frac{R(N)}{N}, \quad (15)$$

where $W^M = \frac{1}{2}\left(A^L + \frac{R(N)}{N}\kappa\right) = \frac{1}{2}\left(A^L + R'(N)\right)$ is the monopsony wage, and $R(N)/N \geq W$ in order for $\Pi \geq 0$.

Obviously, (15) cannot hold if the bargaining strength of the union is very high. In order for (15) to hold, the outcome must be a wage such that labor demand is not binding, that is $W < W^*$. The bargaining strength of the firm γ must be sufficiently high. From (15) it follows that this condition is fulfilled only if

$$\frac{1}{2}\gamma A^L + \frac{R(N)}{N}\left((1-\gamma) + \frac{1}{2}\gamma\kappa\right) < W^*$$

$$\frac{1}{2}\gamma A^L + \frac{W^*}{\kappa}(1-\gamma) + \frac{1}{2}\gamma W^* < W^* \quad (16)$$

$$\gamma\left(W^* - \frac{1}{2}\kappa\left(A^L + W^*\right)\right) > (1-\kappa)W^*,$$

which gives

$$\gamma > \frac{(1-\kappa)W^*}{(1-\kappa)W^* + \frac{1}{2}\kappa\left(W^* - A^L\right)} \equiv \tilde{\gamma}. \quad (17)$$

Lastly, consider the case where both constraints are binding, $\lambda > 0$ and $\mu > 0$. We know that this is possible only for one wage level, namely for $W = W^*$. The relevant question is under what conditions this solution occurs, i.e., under which conditions both $\lambda > 0$ and $\mu > 0$. Inserting (6) into (7) for λ, utilizing that $R'(N) = W^*$ when $\lambda > 0$, yields

$$(1-\gamma)+R''(N)\left(\frac{1-\gamma}{W^*-A^L}-\frac{\gamma N}{R(N)-W^*N}+\mu\phi\right)N-\mu N=0$$

$$(1-\gamma)\left(\frac{1}{\kappa}-1\right)\left(W^*-A^L\right)+\varepsilon\left((1-\gamma)\left(\frac{1}{\kappa}-1\right)W^*-\gamma\left(W^*-A^L\right)\right) \tag{18}$$

$$+\mu\left(\frac{1}{\kappa}-1\right)\left(W^*-A^L\right)\left(\varepsilon\phi W^*-N\right)=0.$$

Utilizing that $N=\phi\left(W-A^L\right)$, this can be written as

$$(1-\gamma)(1-\kappa)\left(W^*-A^L+\varepsilon W^*\right)+\gamma\kappa\varepsilon\left(W^*-A^L\right)+\mu(1-\kappa)\left(\varepsilon W^*-\left(W^*-A^L\right)\right)N=0. \tag{19}$$

In order for $\mu>0$, we must have

$$\mu=\frac{\left((1-\gamma-\kappa)(1+\varepsilon)+\gamma\kappa\right)W^*-\left(1-\gamma-\kappa+\gamma\kappa(1-\varepsilon)\right)A^L}{(1-\kappa)N\left(W^*(1-\varepsilon)-A^L\right)}>0. \tag{20}$$

(20) implies that $\mu>0$ if

$$\left((1-\gamma-\kappa)(1+\varepsilon)+\gamma\kappa\right)W^*-\left(1-\gamma-\kappa+\gamma\kappa(1-\varepsilon)\right)A^L>0$$
$$\gamma\left(\left(\kappa-(1+\varepsilon)\right)W^*+\left(1-\kappa(1-\varepsilon)\right)A^L\right)>(1-\kappa)(1+\varepsilon)W^*-(1-\kappa)A^L, \tag{21}$$

which can be written

$$\gamma>\frac{(1-\kappa)\left(A^L-(1+\varepsilon)W^*\right)}{(1-\kappa)\left(A^L-(1+\varepsilon)W^*\right)-\kappa\varepsilon\left(W^*-A^L\right)}=\hat{\gamma}. \tag{22}$$

In conclusion, (22) must hold in order for the supply constraint to be binding.

Next, we have to undertake the same exercise for λ. Inserting (7) into (6) for μ, utilizing that $R'(N)=W^*$ when $\lambda>0$, yields

$$-\gamma N\left(W^*-A^L\right)+(1-\gamma)\left(R(N)-W^*N\right)-\left(\lambda-\varphi\left((1-\gamma)\frac{1}{N}+\lambda R''(N)\right)\right)\left(R(N)-W^*N\right)\left(W^*-A^L\right)=0 \tag{23}$$

$$-\gamma\left(W^*-A^L\right)+(1-\gamma)\left(\frac{1}{\kappa}-1\right)W^*-\left(\lambda\left(W^*-A^L\right)-(1-\gamma)-\lambda\varepsilon W^*\right)\left(\frac{1}{\kappa}-1\right)W^*=0. \tag{24}$$

In order for $\lambda>0$, we must have

$$\lambda=\frac{2(1-\gamma)(1-\kappa)W^*-\gamma\kappa\left(W^*-A^L\right)}{(1-\kappa)W^*\left(W^*(1-\varepsilon)-A^L\right)}>0. \tag{25}$$

(25) implies that $\lambda > 0$ if

$$2(1-\gamma)(1-\kappa)W^* - \gamma\kappa(W^* - A^L) > 0$$
$$\gamma(2W^*(1-\kappa) + \kappa(W^* - A^L)) < 2(1-\kappa)W^* \tag{26}$$

which implies that

$$\gamma < \frac{W^*(1-\kappa)}{W^*(1-\kappa) + \frac{1}{2}\kappa(W^* - A^L)} = \tilde{\gamma}. \tag{27}$$

Taken together, both constraints bind if (22) and (27) hold simultaneously, that is, $\hat{\gamma} \le \gamma \le \tilde{\gamma}$. If $\hat{\gamma} = \tilde{\gamma}$, this is true for only one value of γ, as for all other possible outcomes. If there is strict inequality, however, both constraints bind for a range of values of γ. From (22) and (27) it follows that $\hat{\gamma} < \tilde{\gamma}$ if

$$\frac{(1-\kappa)(A^L - (1+\varepsilon)W^*)}{(1-\kappa)(A^L - (1+\varepsilon)W^*) - \kappa\varepsilon(W^* - A^L)} < \frac{W^*(1-\kappa)}{W^*(1-\kappa) + \frac{1}{2}\kappa(W^* - A^L)} \tag{28}$$

$$(A^L - (1+\varepsilon)W^*)(W^*(1-\kappa) + \tfrac{1}{2}\kappa(W^* - A^L)) < W^*((1-\kappa)(A^L - (1+\varepsilon)W^*) - \kappa\varepsilon(W^* - A^L))$$
$$\tfrac{1}{2}\kappa(A^L - W^*(1+\varepsilon)) < -\kappa\varepsilon W^*, \tag{29}$$
$$(1-\varepsilon)W^* - A^L > 0$$

which proves that $\hat{\gamma} < \tilde{\gamma}$ because $\varepsilon < 0$ and $W^* > A^L$. Thus, for a range of values of the bargaining power, $\hat{\gamma} \le \gamma \le \tilde{\gamma}$, both the supply and demand constraint hold, and the wage is given by W^* for employment equal to N*.